MR2964857 57N13 57R65 57R67 57R80
Khan, Qayum (1-NDM)
Homotopy invariance of 4-manifold decompositions: connected sums. **(English summary)**
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The author proves that under certain hypotheses the existence and uniqueness, up to h-cobordism, of connected sum decompositions of orientable 4-manifolds is an invariant of homotopy equivalence. A discrete group G is NDL if the π_1-Null Disc Lemma holds [M. H. Freedman and P. Teichner, Invent. Math. **122** (1995), no. 3, 509–529; MR1359602]. A nonempty compact connected topological 4-manifold Z with fundamental group π and orientation character ω has class SES^h_+ if the surgery exact sequence, including actions of the groups in K- and L-theory, holds:

$$\mathcal{N}_{\mathrm{TOP}}(Z\times I)\to L^h_5(\pi,\omega)\to \mathcal{S}^h_{\mathrm{TOP}}(Z)\to \mathcal{N}_{\mathrm{TOP}}(Z)\to L^h_4(\pi,\omega).$$

Main Theorem: Let X be a compact connected topological manifold of dimension 4. (1) Suppose the fundamental group $\pi_1(X)$ is a free product of groups of class NDL. If X is non-orientable, assume $\pi_1(X)$ is 2-torsion free. Then there exists $r\geq 0$ such that the r-th stabilization $X\#r(S^2\times S^2)$ has class SES^h_+.

(2) Suppose X has the homotopy type of a connected sum $X_1\#\cdots\#X_n$ such that each X_i has class SES^h_+. If X is non-orientable, assume that $\pi_1(X)$ is 2-torsion free. Then the homotopy connected sum X has class SES^h_+. Moreover, the induced function is a bijection:

$$\#\colon \prod_{i=1}^n \mathcal{S}^h_{\mathrm{TOP}}(X_i)\to S^h_{\mathrm{TOP}}(X).$$

The proof involves using techniques of S. Weinberger [Israel J. Math. **59** (1987), no. 1, 1–7; MR0923658] to homology split along essential 3-spheres and performing a neck exchange trick to replace homology 3-spheres by 3-spheres [cf. M. H. Freedman and F. Quinn, *Topology of 4-manifolds*, Princeton Math. Ser., 39, Princeton Univ. Press, Princeton, NJ, 1990; MR1201584; M. Kreck, W. Lück and P. Teichner, Comment. Math. Helv. **70** (1995), no. 3, 423–433; MR1340102; B. Jahren and S. Kwasik, Math. Proc. Cambridge Philos. Soc. **140** (2006), no. 2, 245–252; MR2212277]. Critical use is made of the high-dimensional splitting obstruction group [S. E. Cappell, Bull. Amer. Math. Soc. **80** (1974), 1193–1198; MR0356091] vanishing, which was recently proved by Frank Connolly and Jim Davis.

The author also proves results on topological s-rigidity and applies them together with the main theorem to show that with certain hypotheses involving fundamental groups satisfying the Farrell-Jones conjecture in L-theory [F. T. Farrell and L. E. Jones, J. Amer. Math. Soc. **6** (1993), no. 2, 249–297; MR1179537], a connected sum $X:=X_1\#\cdots\#X_n$ is topologically s-rigid. *Terry Lawson*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.