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[Khan, Qayum](#) (1-IN)

Countable approximation of topological G -manifolds, II: linear Lie groups G .

(English summary)

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Generalizing a result by E. G. J. Elfving [Ann. Acad. Sci. Fenn. Math. Diss. No. 108 (1996), 50 pp.; [MR1413841](#)], the author proves the following theorem:

Theorem 1. Let G be a linear Lie group. Let M be a separable metrizable cohomological manifold over the integers. If M is equipped with a Palais proper G -action and the fixed point sets M^H are ANRs for each compact subgroup $H \subset G$, then M is G -homotopy equivalent to a countable proper G -CW complex.

This theorem immediately implies that any topological G -manifold equipped with a Palais proper G -action has the equivariant homotopy type of a countable proper G -CW complex.

{For Part I see [MR3760187](#).}

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.