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Dihedral manifold approximate fibrations over the circle. (English summary)

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When a manifold  $W$  admits a free action of the infinite cyclic group  $C_\infty$ , one expects the quotient  $M$  to fiber over  $S^1$  so that the infinite cyclic covering  $\overline{M}$  induced from the universal covering  $\mathbb{R} \rightarrow S^1$  is  $W$ . When  $\dim(W) > 4$ , work of T. A. Chapman gave this idea a precise formulation, which states that the following conditions (among others) are equivalent: (1)  $W$  is finitely dominated and admits a cocompact, free, discontinuous  $C_\infty$ -action, (2) there is a proper bounded fibration  $W \rightarrow \mathbb{R}$ , (3) there exists a  $C_\infty$ -action on  $W$  and  $C_\infty$ -manifold approximate fibration  $W \rightarrow \mathbb{R}$ , and (4) there exists a manifold approximate fibration  $M \rightarrow S^1$  with corresponding infinite cyclic covering  $\overline{M}$  homeomorphic to  $W$ .

The main results of the paper extend these to corresponding conditions with the infinite dihedral group  $D_\infty$  in the role of  $C_\infty$ . To prove them, the authors employ a considerable amount of machinery of the theory of approximate fibrations and stratified fibrations. In particular, they draw on various techniques and results of Chapman (crediting S. Ferry with independent discovery of many of the same ideas), and of D. S. Coram and P. Duvall. Geometric concepts also come into play; in particular, the authors develop a condition on a triangulated metric space, called finite isometry type, which makes precise the idea that the local geometry of a space has finite variation. The paper is very carefully written, with considerable detail in the arguments and extensive referencing of the needed results from the literature.

{For further information pertaining to this item see [C. B. Hughes and Q. Khan, *Geom. Dedicata* **164** (2013), 395; [MR3054635](#)].}

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