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**On fibering and splitting of 5-manifolds over the circle. (English summary)**

*Topology Appl.* **156** (2008), *no. 2*, 284–299.

The author’s summary is a concise account of the contents of the paper under review: “Our main result is a generalization of Cappell’s 5-dimensional splitting theorem. As an application, we analyze, up to internal s-cobordism, the smoothable splitting and fibering problems for certain 5-manifolds mapping to the circle. For example, these maps may have homotopy fibers which are in the class of finite connected sums of certain geometric 4-manifolds. Most of these homotopy fibers have non-vanishing second mod 2 homology and have fundamental groups of exponential growth, which are not known to be tractable by Freedman-Quinn topological surgery. Indeed, our key technique is topological cobordism, which may not be the trace of surgeries.”

As usual, in order to apply the standard high-dimensional splitting and fibration theorems in dimension 5 it is necessary to introduce all kinds of special hypotheses.

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*

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