MR3718432 57R67 19J05 57N65 57S17
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Free transformations of $S^1 \times S^n$ of square-free odd period. (English summary)
Indiana Univ. Math. J. **66** (2017), *no. 5,* 1453–1482.

B. Jahren and S. Kwasik classified topological free involutions on $S^1 \times S^n$ up to conjugation [Math. Ann. **351** (2011), no. 2, 281–303; MR2836659]. In the present paper the author considers free $\mathbb{Z}_\ell$-actions on $S^1 \times S^n$ up to conjugation. Let $\mathcal{A}^n_\ell$ be the set of conjugacy classes (C_ℓ) in $\mathrm{Homeo}(S^1 \times S^n)$ of cyclic subgroups C_ℓ of order ℓ without fixed points. The author obtains a classification of $\mathcal{A}^n_\ell$ when $\ell > 1$ is square-free odd: If $n = 2k$ ($k > 0$) or $n = 1$, then $\mathcal{A}^n_\ell = \{(T_\ell)\}$, where T_ℓ is the rotation of order ℓ on S^1 and the identity on S^n. For all other $n = 2k - 1$ there is a finite-to-one surjection of the disjoint union of

$$\mathcal{Q}^k_d \times \mathbb{Z}^{(d-1)/2} \times H_0(C_2; \mathrm{Wh}_0(C_d)) \to \mathcal{A}^n_\ell - \{(T_\ell)\},$$

where $\mathcal{Q}^k_d$ is a certain partition of $\mathbb{Z}^\times_d$ and the union is over $1 < d|\ell$. In particular, the set of conjugacy classes of free $\mathbb{Z}_\ell$-actions on $S^1 \times S^{2k-1}$ is countably infinite if $k > 1$. By passage to orbit spaces the author obtains a bijection of $\mathcal{A}^{2k-1}_\ell - \{(T_\ell)\}$ to the union of the sets of closed topological manifolds homotopy equivalent to $S^1 \times L^{2k-1}_{d,q}$, where the union is over $1 < d|\ell$ and $[q] \in \mathcal{Q}^k_d$. The techniques use surgery theory, homotopy theory, and a careful study of h-cobordisms. Along the way the author obtains a homotopy classification of orbit spaces and a classification of h-cobordism types. For example: (1) If $S^1 \times S^n$ is a regular cyclic ℓ-fold cover of a topological space M, for $n \geq 1$ and ℓ odd > 1, then M is homotopy equivalent to $S^1 \times S^n$ or $S^1 \times L^n_{d,q}$ with $d|\ell$. (2) If M and X are closed connected topological manifolds of dimension $n \geq 4$ that are homotopy equivalent (and if $n = 4$, $\pi_1(X)$ is good in the sense of Freedman and Quinn), then $SI(M) \cong SI(X)$ as subgroups of $\mathrm{Wh}_1(\pi_1 M) \cong \mathrm{Wh}_1(\pi_1 X)$, where $SI(M)$, $SI(X)$ consist of the Whitehead torsions of strongly inertial h-cobordisms. *Wolfgang H. Heil*

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