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Connolly, Frank [Connolly, Francis X.] (1-NDM);
Davis, James F. [Davis, James Frederic] (1-IN); **Khan, Qayum** (1-STL)

Topological rigidity and H_1-negative involutions on tori. (English summary)

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In the present paper the authors begin a systematic study of the Borel-Quinn conjecture [F. Quinn, in *Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Berkeley, Calif., 1986)*, 598–606, Amer. Math. Soc., Providence, RI, 1987; MR0934261] and closely related questions of equivariant and topological rigidity for a discrete cocompact group of isometries. They are motivated by the rigidity theorem of Farrell and Jones concerning the classification of proper actions of discrete, torsion-free cocompact isometry groups Γ on contractible manifolds and the recent work of Cappell on the UNil groups. Moreover, the authors draw on a remarkable, rich and fertile literature concerning these and related questions, much of the work being their own.

Summary: "We show, for $n \equiv 0, 1 \pmod 4$ or $n = 2, 3$, there is precisely one equivariant homeomorphism class of C_2 manifolds (N^n, C_2) for which N^n is homotopy equivalent to the n-torus and $C_2 = \{1, \sigma\}$ acts so that $\sigma_*(x) = -x$ for all $x \in H_1(N)$. If $n \equiv 2, 3 \pmod 4$ and $n > 3$ we show there are infinitely many such C_2-manifolds. Each is smoothable with exactly 2^n fixed points.

"The key technical point is that we compute, for all $n \geq 4$ the equivariant structure set $\mathcal{S}_{\mathrm{TOP}}(\mathbb{R}^n, \Gamma_n)$ for the corresponding crystallographic group Γ_n in terms of the Cappell UNil-groups arising from its infinite dihedral subgroups." *Ronald M. Dotzel*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.