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Topological rigidity and actions on contractible manifolds with discrete singular set. (English summary)
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In the paper under review, the authors study a version of the equivariant Borel conjecture, which is a version of equivariant rigidity. For a discrete group Γ, let $\mathcal{S}(\Gamma)$ be the set of equivariant homeomorphism classes of pairs $[M,\Gamma]$ of contractible topological manifolds with an effective Γ action. The rigidity result states that $\mathcal{S}(\Gamma)$ is trivial. When Γ is a torsion-free group that acts on a complete nonpositively curved manifold by isometries, Farrell and Jones showed that the structure set has only one element. The main result is that if the group Γ satisfies the conditions stated in the paper, then the structure set is given as a direct sum of UNil-groups. In particular, if the virtual cohomological dimension n of Γ satisfies $n\equiv 0,1 \mod 4$ or if Γ has no elements of order 2, then the structure set has a single element. The assumptions are very natural.

We assume that $vcd(\Gamma)\geq 4$. There is a series of reductions of the structure set that helps in the final calculation. For a cocompact Γ-manifold, let $\mathcal{S}(X,\Gamma)$ be the structure set of Γ-homotopy equivalences $f\colon M\to X$, where M is a cocompact Γ-manifold. Also, let $\mathcal{S}^{iso}(X,\Gamma)$ be the isovariant structure set. The assumptions are that Γ is a virtually torsion-free group and that the normalizers of the finite subgroups are finite. Then any contractible, effective, cocompact Γ-manifold M is a model of an $E_{\mathfrak{fin}}\Gamma$ space, i.e. the fixed point sets of finite subgroups are contractible and those of the infinite groups are empty. Also, it is shown that fixed point sets are points. These actions are called pseudo-free. The reductions are that, under the assumptions stated above, the forgetful maps $\mathcal{S}^{iso}(X,\Gamma)\to\mathcal{S}(X,\Gamma)\to\mathcal{S}(\Gamma)$ are bijections.

For the next reduction, the authors impose more conditions on Γ. It is assumed that there is a contractible, cocompact, effective Γ-manifold X so that X_{free}/Γ has the homotopy type of a finite CW-complex. Also, it is assumed that every dihedral group is contained in a unique maximal dihedral group and that Γ satisfies the K- and L-theory Farrell-Jones Conjecture. Then the reduction is that the forgetful map

$$\mathcal{S}^h_{TOP}(\overline{X},\partial\overline{X})\to\mathcal{S}^{iso}(X,\Gamma)$$

is a bijection. Here $\overline{X}$ is a compact manifold with boundary with interior X_{free}/Γ. The structure set $\mathcal{S}^h_{TOP}(\overline{X},\partial\overline{X})$ is the structure set of homotopy equivalences of pairs with equivalence relation given by h-cobordisms.

The last structure set is bijective to Ranicki's algebraic structure set $\mathcal{S}^h_{n+1}(\overline{X},\partial\overline{X})$. Ranicki shows that the last structure set embeds into the periodic structure set $\mathcal{S}^{per,h}_{n+1}(\overline{X},\partial\overline{X})$ with cokernel being a homology group. The next reduction is that the forgetful map

$$\mathcal{S}^{per,h}_{n+1}(\overline{X},\partial\overline{X})\to\mathcal{S}^{per,-\infty}_{n+1}(\overline{X},\partial\overline{X})$$

is a bijection. Furthermore, it is proved that

$$\mathcal{S}^{per,-\infty}_{*}(\overline{X},\partial\overline{X})\cong H^{\Gamma}_{*}(E_{\mathfrak{all}}\Gamma,E_{\mathfrak{fin}}\Gamma;\underline{\mathbf{L}}).$$

The last homology group is the direct sum of the corresponding homology groups for the maximal infinite dihedral groups. In another paper [Geom. Topol. **18** (2014), no. 3, 1719–1768; MR3228461] the authors proved that the last homology groups are isomorphic to UNil-groups.

The second main result is that if Γ satisfies the first two conditions and the K-theoretic Farrell-Jones Conjecture, then if $vcd(\Gamma) \geq 6$, and W is a locally flat isovariant cobordism between two Γ-manifolds M and M' of type $E_{\mathsf{fin}}\Gamma$, then W is the product cobordism and the two manifolds are Γ-homeomorphic. Since in the orbit space, W, induces an h-cobordism of manifold homotopically stratified spaces, the proof uses the corresponding theory.

At the end, the authors present examples of groups that satisfy the conditions, for example $\Gamma = \mathbb{Z}^n \times_\rho C_m$, where the cyclic group C_m action on $\mathbb{Z}^n$ is free on $\mathbb{Z}^n - \{0\}$. For the second example, assume that there is a pseudo-free action of a finite group G on a closed PL-manifold. Let $h(K)$ be the hyperbolization of K, which is also a G-space, and Γ be the group of homeomorphisms of the universal cover of $h(K)$ that cover the elements of G. Then Γ satisfies the conditions of the main theorem. *Stratos Prassidis*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.