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Reduction of UNil for finite groups with normal abelian Sylow 2-subgroup. (English summary)
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Let $\mathrm{UNil}_n^{\langle -\infty\rangle}(R;R,R)$ denote Cappell's ultimate unitary nilpotent L-group of a ring R with involution. (Assume trivial orientation character throughout.) Let F be a finite group with a normal abelian Sylow 2-subgroup S. The author shows that the map

$$\mathrm{UNil}_n^{\langle -\infty\rangle}(\mathbb{Z}[S];\mathbb{Z}[S],\mathbb{Z}[S])_{F/S}\longrightarrow \mathrm{UNil}_n^{\langle -\infty\rangle}(\mathbb{Z}[F];\mathbb{Z}[F],\mathbb{Z}[F])$$

defined on the quotient of $\mathrm{UNil}_n^{\langle -\infty\rangle}(\mathbb{Z}[S];\mathbb{Z}[S],\mathbb{Z}[S])$ by the action of the group F/S, induced by the inclusion, is an isomorphism. It follows that if F is of odd order, then the inclusion of the trivial subgroup induces an isomorphism

$$\mathrm{UNil}_n^{\langle -\infty\rangle}(\mathbb{Z};\mathbb{Z},\mathbb{Z})\cong \mathrm{UNil}_n^{\langle -\infty\rangle}(\mathbb{Z}[F];\mathbb{Z}[F],\mathbb{Z}[F]),$$

because in that case $S=1$. The group $\mathrm{UNil}_n^{\langle -\infty\rangle}(\mathbb{Z};\mathbb{Z},\mathbb{Z})$ has been computed in [F. X. Connolly and J. F. Davis, Geom. Topol. **8** (2004), 1043–1078 (electronic); MR2087078; M. Banagl and A. A. Ranicki, Adv. Math. **199** (2006), no. 2, 542–668; MR2189218].

To prove his result, the author uses the Connolly-Ranicki correspondence

$$\mathrm{UNil}_n^{\langle -\infty\rangle}(R;R,R)\cong NL_n^{\langle -\infty\rangle}(R),$$

with

$$NL_n^{\langle -\infty\rangle}(R)=\ker(L_n^{\langle -\infty\rangle}(R[x])\to L_n^{\langle -\infty\rangle}(R)),$$

and establishes 2-hyperelementary induction for the Mackey functor $\mathcal{N}(F)= NL_n^{\langle -\infty\rangle}(\mathbb{Z}[F])$ on finite groups F. This induction leads to a hyperelementary reduction theorem stating that if $S\subset F$ is a normal subgroup and for all 2-hyperelementary subgroups $H\subset F$ the inclusion induces an isomorphism $\mathcal{N}(H\cap S)\cong\mathcal{N}(H)$, then $\mathcal{N}(S)_{F/S}\cong\mathcal{N}(F)$. Vanishing theorems and homological analysis of cyclotomic number rings help to control the odd-order information.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.